EFFECT OF TRANSFORMER LEAKAGE INDUCTANCE ON
THE THREE PHASE CAPACITIVE INPUT RECTIFIER

James O'Loughlin
Douglas Larson

Air Force Weapons Laboratory/ARAY
Kirtland Air Force Base
NM 87117

Summary

The characteristics of three phase capacitive input rectifiers which include the effects of transformer leakage inductance have been generated by computer analysis and reduced to a set of design curves. The inclusion of sufficient inductance leads to a three phase commutation mode which in general cannot be accounted for by a superposition of single phase design curves[1]. The results presented fill a long standing gap in the literature with respect to a practical set of data on the design of three phase rectifiers with capacitive input filters.

Introduction

Single phase capacitive input filter analysis including inductive reactance was first published in 1934 by R. L. Freeman[2]. In 1943 the well-known "Schade's Curves" were published by O. H. Schade[3] for single phase capacitive input filters but did not include the effects of inductive reactance. "Schade's Curves" were updated to include inductive (transformer) reactance by J. P. O'Loughlin and S. L. West[1] in 1981. Three phase performance can be determined from single phase curves provided that the current conduction angles do not exceed 60° (i.e., overlap). In the case with zero inductive reactance it can be shown that for $\omega RC > \sqrt{3}$ the conduction angle for each of the six current conduction intervals per cycle will be less than 60°. The presence of inductive (transformer) reactance tends to increase the conduction angle and a so-called critical value of inductive reactance exists as a function of ωRC. The term "critical leakage inductance" is taken after the inductive input filter term "critical inductance". In both cases it refers to the minimum value of inductance required to just maintain a continuous flow of current out of the rectifier assembly. As a point of information it should be noted that the inductor in an inductive input filter is on the load side of the rectifier. Whereas the inductance under consideration with the capacitive input filter is normally the transformer leakage inductance, see Fig. 1. Once the critical leakage inductance is exceeded the circuit can no longer be analyzed by the superposition of currents derived from the single phase curves because of the overlap or commutation between phases. It is the purpose of this paper to present data in particular which relates to operation with more than critical leakage inductance. It is emphasized that in any capacitive input rectifier circuit the amount of leakage inductance typically present in a transformer, i.e., in the order of a few percent (per unit), is enough to vastly reduce the peak and RMS currents compared to the zero inductance behavior. This effect occurs regardless of whether critical leakage inductance has been exceeded or not and must be accounted for to accurately determine component ratings and circuit performance.

Method of Analysis

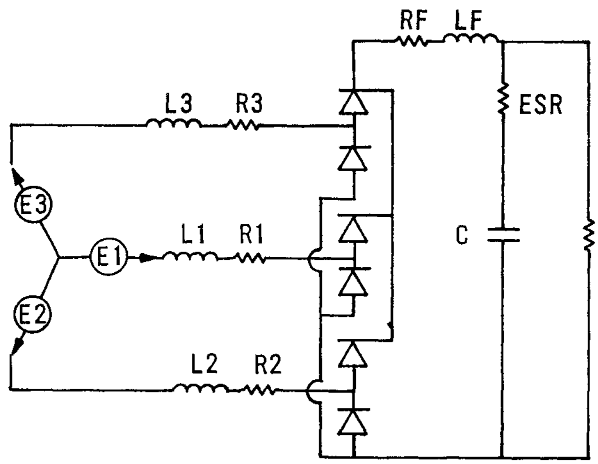

Figure 1

THREE PHASE RECTIFIER

The circuit shown in Fig 1 is the basis of the data presented in this paper. A computer program was written to solve the circuit parameters as a function of finite time increments. The program is built upon the four basic modes or configurations of the circuit as determined by the rectifier states. The null mode, i.e., all rectifiers non-conducting, is unique but all other modes are degenerate as permutations of the phase rotation of the three phase source. Thus the program is based upon four circuit equations related to the four basic modes and simply changes tags to accommodate the rotationally degenerate modes. With the degenerate modes included there are a total of 13 as shown in Fig 2. The program operates by checking the state of the rectifiers at each time step and then selects the appropriate mode equations to calculate the updated circuit currents and voltages. In order

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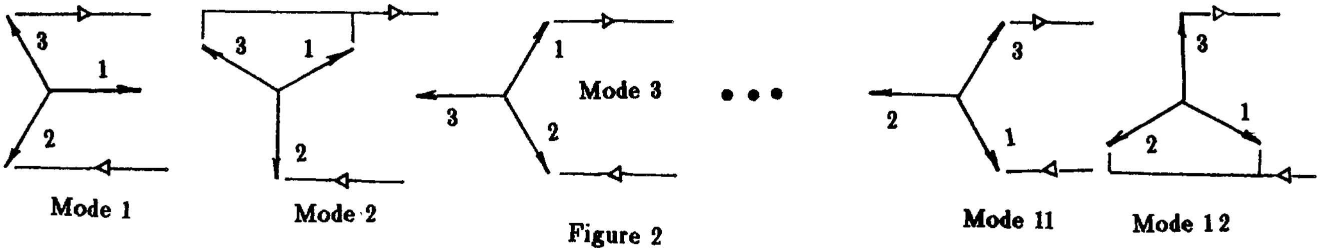

Figure 2

Three Phase Rectifier Conduction Modes

To generate data for a set of design curves the program run time was selected to insure the circuit has settled into steady state operation before the peak, average and RMS values were calculated for usable output. The circuit shown is a simplified version of a more complicated circuit which was originally used. The original circuit and program contains a delta-wye transformer between the source and rectifier and also includes primary and secondary leakage inductances and resistances, and mutual inductances between the leakage inductances. It is not practical, for the purposes of this paper, to include the effects of mutual inductances or resistance for at least two reasons. First the mutual between leakages is not in general symmetric nor is it usually known or available from transformer data sheets. Second, the number of curves required to present the results and the amount of computer time required to generate them becomes prohibitive. Some estimates of typical mutual inductances and winding resistances were made and the effects proved to be second order. Also for the same practical reasons the analysis was restricted to a balanced system. It is, however, recommended in critical applications that proper consideration be given to mutual coupling between the leakage inductances by running a suitably detailed circuit model analysis for the particular configuration.

As previously mentioned, it can be shown by simple analysis that a critical value of $\omega RC = \sqrt{3}$ exists in the three phase rectifier circuit. Since usually the line frequency ω and load resistance R are not a matter of choice, the only option to consider is the filter capacitance C. Thus one could define a critical value of capacitance as:

$$C\ (\text{critical}) = \sqrt{3}/(\omega R).$$

It is significant that values of capacitance less than critical do not provide any ripple filtering in the three phase rectifier. This is not the case with the single phase rectifier where any value of capacitance will result in a corresponding reduction in ripple voltage.

The three phase rectifier also has a critical value of transformer winding resistance, R_T(critical). Under the conditions of low ripple, i.e., $\omega RC \geq 10$, when the critical value of winding resistance is exceeded the rectifier assembly will deliver continuous current, i.e., the conduction angle per phase will be 60^o or more. The value of critical winding resistance referred to the secondary leg is:

$$R_T(\text{critical}) = (\sqrt{3}/\pi - 1/2)\ R = .051328R$$

This value is somewhat higher than one would find in a typical transformer which is usually in the one to three percent range.

The authors were unable to derive a closed algebraic expression for the value of critical leakage inductance due to the transcendental equations which crept into the analysis. The critical leakage was therefore evaluated by the computer model. The critical leakage inductance thus determined is plotted as reactance normalized to the load resistance vs ωRC in Fig. 3.

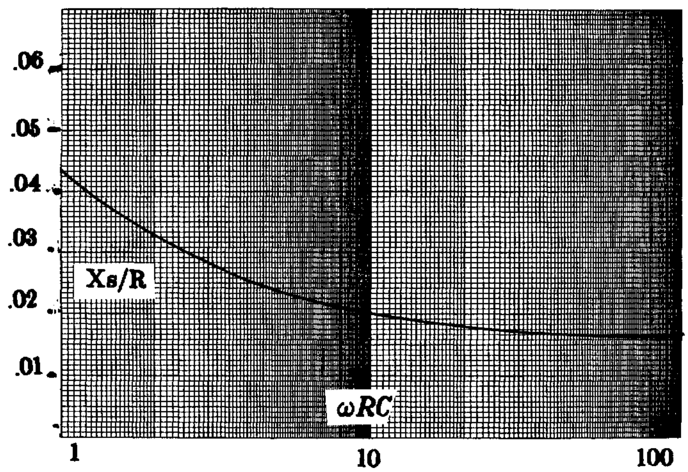

Figure 3

Normalized Critical Reactance Per Phase

vs

ωRC

Results

The computer program for the circuit in Fig. 1 was used to generate design performance data under the following conditions. The winding resistances were taken to be zero, and a balanced three phase system was used. The amount of leakage reactance expressed as a percentage of the load resistance was swept over a range of .1% to 10% in five steps. The independent variable was taken (after Schade) as ωRC and was swept over the range of .1 to 1000. The results are plotted in Figs. 4, 5, and 6 as normalized, d.c. output voltage, RMS line current and peak line current. Figs. 7 and 8 show the percentage peak and RMS ripple voltage. The resonance effect which occurs for values of X_s/R_L less than $\approx$ 3% is exaggerated somewhat due to the fact that the winding resistance has been neglected. This is not of concern in nearly all practical situations due to the fact that transformer leakage reactances typically run 3 to 7 percent.

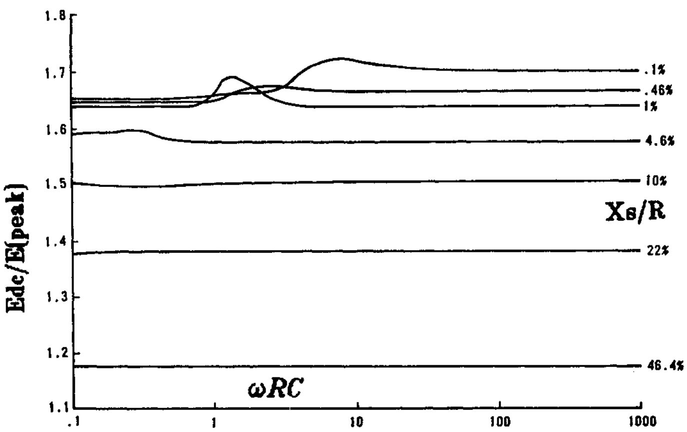

Figure 4
Normalized D.C. Load Voltage
vs ωRC
For Xs/R=.1,.46,1,4.6,10,22,46.4%

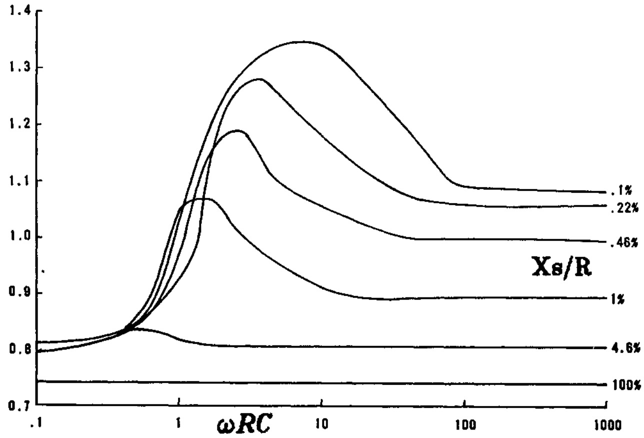

Figure 5
Normalized RMS Line Current vs ωRC

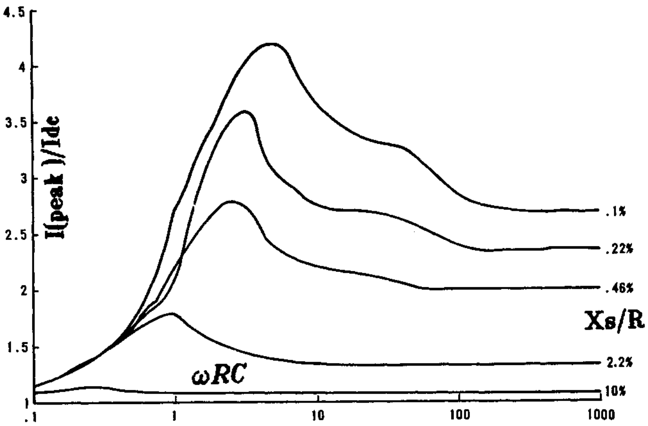

Figure 6
Normalized Peak Line Current vs ωRC

For Xs/R=.1,.2,.46,2.2,10%

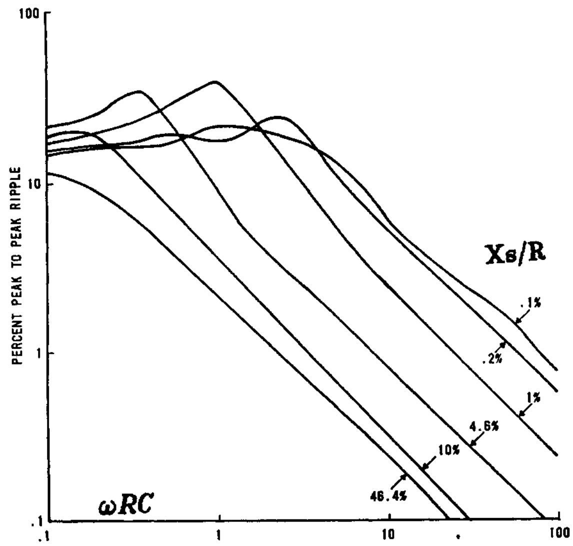

Figure 7
Percent Peak to Peak Ripple vs ωRC
For Xs/R=.1,.2,1,4.6,10,46.4%

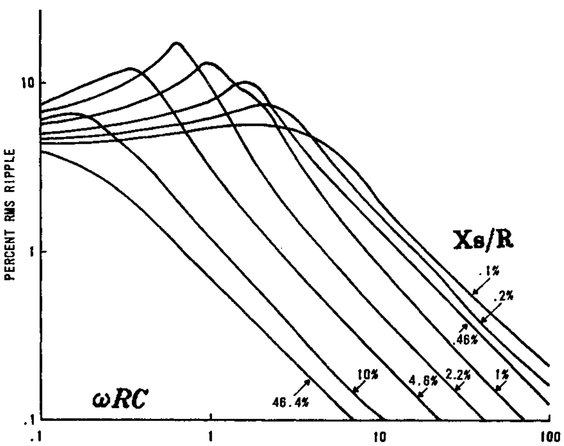

Figure 8
Percent RMS Ripple vs ωRC

For Xs/R=.1,.2,.46,1,2.2,4.6,10,46.4%

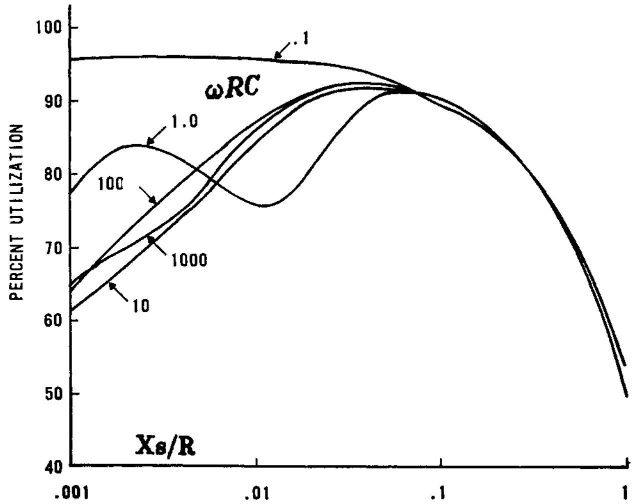

Figure 9
Transformer Utilization Ratio vs Xs/R
For ωRC =.1,1,10,100,1000

Transformer utilization is almost always an important consideration. Utilization relates the usable d.c. output power to the required transformer volt-ampere (VA) rating. A low utilization occurs when the ratio of RMS to d.c. current is high. The effect of leakage resistance and ωRC on utilization is shown in Fig. 9. The results show a utilization over the range of practical interest ($.02 < Xs/RL < .1$) which is in the order of 90%. This is much better than the estimates as low as 15% one typically obtaines if the leakage reactance is neglected. In order to evaluate the accuracy of the computer model and also assess the effect of neglecting resistance, a real circuit was tested. The measured results compared to the results given by the computer model with and without the circuit resistance are given in Table I.

Table I

Test Results Compared to Computer Model

PARAMETER	MEASURED DATA	WITH RESISTANCE	WITHOUT RESISTANCE
D.C. Load Voltage	228	228.08	236.85
Normalized D.C.	.767	.767	.797
P-P Ripple %	21.1	22.2	25.4
RMS Ripple %	7.0	7.82	8.85
I_{RMS}/I_{DC}	.962	.963	1.011
I_{PEAK}/I_{DC}	1.993	1.825	1.963
Utilization	.783	.785	.786
Conduction Angle	61.56^{o}	61.02^{o}	60.12
Commutation Angle	0^{o}	1.8^{o}	0^{o}

When the computer model includes the resistance the agreement is within the limits of measurement, and with resistance neglected the error is at worst 3.5%. The actual values used in the circuit were: R_L = 233.1 ohms, C = 15μfd, transformer inductance per line = 2.09 mh, transformer resistance per line = 4.3 ohms, and the operating frequency of 60 HZ.

Conclusions

The computer model with which the design curves given in this paper were generated has been tested against actual circuits and found to be in agreement within 3.5%. Three phase capacitive input rectifiers can be expected to achieve in the order of 90% transformer utilization in most typical applications.

References

1. J. P. O'Loughlin and S. L. West, "Effect of Transformer Leakage Inductance on Capacitor Input Filter", Digest of Technical Papers, 3rd International Pulsed Power Conference, 1981, pp 491-4, Lib. Congress 81-81007, IEEE 81CH1662-6.

2. R. L. Freeman, "Analysis of Rectifier-Filter Circuits", Doctoral Thesis, Stanford Univ. Library, 1934.

3. O. H. Schade, "Analysis of Rectifier Operation", Proceedings of the IRE, July 1943, pp 341-361.

LIST OF SYMBOLS

C	=Filter capacitance (farads)
Edc	=Average voltage of capacitor C (volts)
E(peak)	=Peak source voltage (volts) Fig.1, E1=E(peak)*Sin(ωt)
R	=Load resistance (ohms)
Rt	=Resistance (transformer) per phase Fig.1 R1,R2,R3 (ohms)
ω	=Source frequency (radians/sec)
Xs	=Reactance per phase (ohms) Fig.1 ωL1,ωL2,etc.